

5/6/16 "Wisdom begins in wonder."-Socrates

HW: "Arithmetic Series" packet HW #1-7 and try #8

AIM: What is an Arithmetic Series?

Exercise #1: Given the arithmetic sequence defined by $a_1 = -2$ and $a_n = a_{n-1} + 5$, then which of the following is the value of $S_5 = \sum_{i=1}^5 a_i$?

(1) 32

(3) 25

(2) 40

(4) 27

$$-2 + 3 + 8 + 13 + 18$$

40

$$a_1 = -2$$

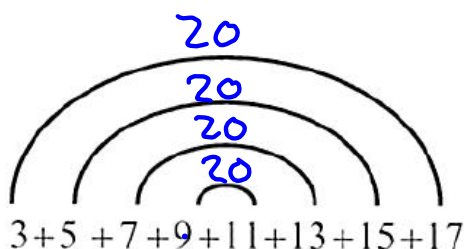
$$a_2 = -2 + 5 = 3$$

$$a_3 = 3 + 5 = 8$$

$$a_4 = 8 + 5 = 13$$

$$a_5 = 13 + 5 = 18$$

Exercise #2: Consider the arithmetic sequence defined by $a_1 = 3$ and $a_n = a_{n-1} + 2$. The series, based on the first eight terms of this sequence, is shown below. Terms have been paired off as shown.



(a) What does each of the paired off sums equal? 20

(b) Why does it make sense that this sum is constant?

$\xrightarrow{+2}$ $\xleftarrow{-2}$ every pair

(c) How many of these pairs are there? Four

(d) Using your answers to (a) and (c) find the value of the sum using a multiplicative process.

$$4 \times 20 = \boxed{80}$$

(e) Generalize this now and create a formula for an arithmetic series sum based only on its first term, a_1 , its last term, a_n , and the number of terms, n .

$$\text{Sum} = \frac{n(a_1 + a_n)}{2}$$

a_1 = first term

a_n = last term

n = number of terms

$$\begin{array}{r}
 1 + 2 + 3 + 4 \dots\dots\dots 1000 \\
 1000 + 999 + 998 + 997 \dots\dots\dots 1 \\
 \hline
 1001 \quad 1001 \quad 1001 \quad 1001 \quad \dots \quad 1001
 \end{array}$$

$$\frac{1000(1001)}{2} = 500,500$$

$$\frac{\text{Number of terms (first + last)}}{2} = \text{Sum}$$

SUM OF AN ARITHMETIC SERIES

Given an arithmetic series with n terms, $\{a_1, a_2, \dots, a_n\}$, then its sum is given by:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Exercise #3: Which of the following is the sum of the first 100 natural numbers?

(1) 5,000

(3) 10,000

(counting numbers)

(2) 5,100

(4) 5,050

$$\text{Sum} = \frac{100}{2}(1+100)$$

$$\begin{aligned} \text{first} &= 1 \\ \text{last} &= 100 \\ \text{total terms} &= 100 \end{aligned}$$

Exercise #4: Find the sum of each arithmetic series described or shown below.

(a) The sum of the sixteen terms given by:

$$\text{sum} = \frac{n}{2}(a_1 + a_n)$$

$$-10 - 6 + -2 + \dots + 46 + 50$$

$$\text{Sum} = \frac{16}{2}(-10 + 50)$$

$$\boxed{\text{Sum} = 320}$$

(b) The first term is -8 , the common difference, d , is 6 and there are 20 terms

$$\begin{aligned} a_1 &= -8 \\ a_n &= 106 \\ n &= 20 \end{aligned}$$

$$\text{Sum} = \frac{20}{2}(-8 + 106)$$

$$\boxed{\text{Sum} = 980}$$

$$\begin{aligned} a_n &= a_1 + d(n-1) \\ &= -8 + 6(20-1) \\ &= 106 \end{aligned}$$

(c) The last term is $a_{12} = -29$ and the common difference, d , is -3 .

$$\begin{aligned} a_n &= -29 \\ n &= 12 \\ a_1 &= 4 \end{aligned}$$

$$\text{Sum} = \frac{12}{2}(4 + -29)$$

$$\boxed{\text{Sum} = -150}$$

$$\begin{aligned} a_n &= a_1 + d(n-1) \\ -29 &= a_1 + (-3)(12-1) \\ -29 &= a_1 - 33 \\ +33 & \quad +33 \\ \hline 4 &= a_1 \end{aligned}$$

(d) The sum $5 + 8 + 11 + \dots + 77$.

Exercise #5: Kirk has set up a college savings account for his son, Maxwell. If Kirk deposits \$100 per month in an account, increasing the amount he deposits by \$10 per month each month, then how much will be in the account after 10 years?

